PRACTICE PAPER 1*

Time allowed : 2 hours **Maximum marks : 40**

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 8 marks and Part-B carries 32 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 4 MCQs.*
3. *Section-II contains 1 case study-based questions.*

Part - B :

1. *It consists of four Sections-III, IV, V and VI.*
2. *Section-III comprises of 5 questions of 1 mark each.*
3. *Section-IV comprises of 4 questions of 2 marks each.*
4. *Section-V comprises of 3 questions of 3 marks each.*
5. *Section-VI comprises of 2 questions of 5 marks each.*
6. *Internal choice is provided in 1 question of Section-III, 1 question of Section-IV, 1 question of Section-V and 2 questions of section-VI. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. The value of $\int \frac{\sqrt{x}}{\sqrt{x^2 - x}} dx$ is

(a) $2\sqrt{x+1} + C$ (b) $2\sqrt{x-1} + C$ (c) $\sqrt{x+1} + C$ (d) none of these

2. Solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$ is

(a) $\log|1+y| = x + \frac{x^3}{7} + C$ (b) $\log|1-y| = x + \frac{x^3}{3} + C$

(c) $\log|1+y| = x - \frac{x^2}{2} + C$ (d) none of these

3. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{3}{\sqrt{6}}$ (c) $\frac{4}{\sqrt{6}}$ (d) $\frac{5}{\sqrt{6}}$

4. What is the degree of the differential equation $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$?

(a) 1 (b) 2 (c) 3 (d) 4

**The paper is for practice purpose. CBSE has yet not released the official sample paper. So, the pattern is suggestive only. For latest information visit www.cbse.gov.in.*

Section - II

Case study-based question is compulsory. Attempt any 4 sub parts. Each sub-part carries 1 mark.

5. A rumour on whatsapp spreads in a population of 5000 people at a rate proportional to the product of the number of people who have heard it and the number of people who have not. Also, it is given that 100 people initiate the rumour and a total of 500 people know the rumour after 2 days.

Based on the above information, answer the following questions.

(i) If $y(t)$ denote the number of people who know the rumour at an instant t, then maximum value of $y(t)$ is

(a) 500 (b) 100 (c) 5000 (d) none of these

(ii) $\frac{dy}{dt}$ is proportional to

(a) $(y - 5000)$ (b) $y(y - 500)$ (c) $y(500 - y)$ (d) $y(5000 - y)$

(iii) The value of $y(0)$ is

(a) 100 (b) 500 (c) 600 (d) 200

(iv) The value of $y(2)$ is

(a) 100 (b) 500 (c) 600 (d) 200

(v) The value of y at any time t is given by

(a) $y=\frac{5000}{e^{-5000kt}+1}$ (b) $y=\frac{5000}{1+e^{5000kt}}$

(c) $y=\frac{5000}{49e^{-5000kt}+1}$ (d) $y=\frac{5000}{49(1+e^{-5000kt})}$

PART - B

Section - III

6. Evaluate : $\int_{\pi/6}^{\pi/4}(\sec^2 x+\operatorname{cosec}^2 x)\,dx$

7. Find the order of the differential equation $x+\left(\frac{dy}{dx}\right)=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

OR

Find the solution of the differential equation $\frac{dy}{dx}=x^3 e^{-2y}$.

8. Find the equation of a plane with intercepts 2, 3 and 4 on the X, Y and Z-axes respectively.

9. If $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then find $P(A \mid B)$.

10. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Section - IV

11. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$, find $\vec{a}\times\vec{b}$ and $|\vec{a}\times\vec{b}|$.

12. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

OR

Find the cartesian equation of the plane passing through a point having position vector $2\hat{i}+3\hat{j}+4\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-2\hat{k}$.

13. If A and B are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then find P(not A and not B).

14. A box contains N coins, of which m are fair and the rest are biased. The probability of getting head when a fair coin is tossed is $\frac{1}{2}$, while it is $\frac{2}{3}$ when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. Find the probability that the coin drawn is fair.

Section - V

15. Evaluate : $\int \frac{x^2+9}{x^4+81}\,dx$

OR

Evaluate : $\int x^2 \sin 2x\,dx$

16. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$.

17. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

Section - VI

18. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then find the value of $a^2 + b^2 + c^2 + 2abc$.

OR

Find the equation of the plane passing through the points (2, 2, –1) and (3, 4, 2) and parallel to the line whose direction ratios are 7, 0, 6.

19. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

OR

Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x=\sqrt{y}$ and y-axis.

ANSWERS

1. (b) : Let $I=\int\frac{\sqrt{x}}{\sqrt{x^2-x}}dx$

$=\int\frac{\sqrt{x}}{\sqrt{x(x-1)}}dx=\int\frac{dx}{\sqrt{x-1}}=2\sqrt{x-1}+C$

2. (c) : We have $\frac{dy}{dx}=(1-x)(1+y)$

$\Rightarrow \frac{dy}{1+y}=(1-x)dx$

$\Rightarrow \int\frac{dy}{1+y}=\int(1-x)dx \Rightarrow \log|1+y|=x-\frac{x^2}{2}+C$

3. (d) : We have, $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\vec{a}\cdot\vec{b}=(2\hat{i}-\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})=2+2+1=5$

$|\vec{b}|=\sqrt{1^2+(-2)^2+1^2}=\sqrt{1+4+1}=\sqrt{6}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ is $\frac{5}{\sqrt{6}}$.

4. (a): Since greatest power of highest order derivative is 1, therefore degree of the given differential equation is 1.

5. (i) (c) : Since, size of population is 5000.
$\therefore$ Maximum value of $y(t)$ is 5000.

(ii) (d) : Clearly, according to given information,

$\frac{dy}{dt}=ky(5000-y)$, where k is the constant of proportionality.

(iii) (a) : Since, rumour is initiated with 100 people.
$\therefore$ When $t = 0$, then $y = 100$
Thus $y(0) = 100$

(iv) (b) : Since, rumour is spread in 500 people, after 2 days.
$\therefore$ When $t = 2$, then $y = 500$.
Thus, $y(2) = 500$

(v) (c): We know that, when $t = 0$, then $y = 100$
This condition is satisfied by option (c) only.

6. $\int_{\pi/6}^{\pi/4}(\sec^2 x+\text{cosec}^2 x)dx=[\tan x-\cot x]_{\pi/6}^{\pi/4}$

$=(1-1)-\left(\frac{1}{\sqrt{3}}-\sqrt{3}\right)=\frac{2}{\sqrt{3}}$.

7. We have, $x+\left(\frac{dy}{dx}\right)=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Highest order derivative is $\frac{dy}{dx}$. So, its order is 1.

OR

We have, $\frac{dy}{dx}=x^3e^{-2y} \Rightarrow e^{2y}\,dy = x^3dx$

On integrating, we get $\frac{e^{2y}}{2}=\frac{x^4}{4}+C'$

$\Rightarrow 2e^{2y} = x^4 + C$, where $C = 4C'$

8. As the plane has intercepts 2, 3 and 4 on X, Y and Z axes respectively.
$\therefore$ The required equation of the plane is

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1 \Rightarrow 6x + 4y + 3z = 12$

9. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\therefore P(A\cap B)=\frac{3}{8}+\frac{5}{8}-\frac{3}{4}=1-\frac{3}{4}=\frac{1}{4}$

$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{1/4}{5/8}=\frac{2}{5}$.

10. We know that, the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab sq. units.

11. Given, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&-1\\1&2&3\end{vmatrix}$

$=(9+2)\hat{i}-(6+1)\hat{j}+(4-3)\hat{k}=11\hat{i}-7\hat{j}+\hat{k}$.

$\therefore |\vec{a}\times\vec{b}|=\sqrt{(11)^2+(-7)^2+(1)^2}=\sqrt{171}$

12. Vector equation of the line passing through (1, 2, –1) and parallel to the line
$5x - 25 = 14 - 7y = 35z$

i.e., $\frac{x-5}{1/5}=\frac{y-2}{-1/7}=\frac{z}{1/35}$ or $\frac{x-5}{7}=\frac{y-2}{-5}=\frac{z}{1}$

is $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(7\hat{i}-5\hat{j}+\hat{k})$

OR

Here, $(x_1, y_1, z_1) = (2, 3, 4)$, $a = 2$, $b = 1$, $c = -2$
Cartesian equation is
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
$\Rightarrow 2(x - 2) + 1(y - 3) - 2(z - 4) = 0$
$\Rightarrow 2x - 4 + y - 3 - 2z + 8 = 0$
$\Rightarrow 2x + y - 2z = -1$

13. Here, $P(A) \cdot P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} = P(A \cap B)$

$\Rightarrow$ Events A and B are independent.

$\Rightarrow$ Events $\bar{A}$ and $\bar{B}$ are also independent.

Now, $P(\bar{A} \cap \bar{B}) = P(\bar{A})\, P(\bar{B})$

($\because$ $\bar{A}$ and $\bar{B}$ are independent events)

= (1 – $P(A)$)(1 – $P(B)$)

$$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

14. Let E be the event that the coin tossed twice shows first head and then tail and F be the event that the coin drawn is fair.

$$P(F/E) = \frac{P(F) \cdot P(E/F)}{P(F) \cdot P(E/F) + P(\bar{F})\, P(E/\bar{F})}$$

$$= \frac{\frac{m}{N} \cdot \frac{1}{2} \cdot \frac{1}{2}}{\frac{m}{N} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{N-m}{N} \cdot \frac{2}{3} \cdot \frac{1}{3}} = \frac{m/4}{m/4 + 2(N-m)/9}$$

$$= \frac{9m}{m + 8N}.$$

15. Let $I = \int \frac{x^2 + 9}{x^4 + 81}\, dx \Rightarrow I = \int \frac{1 + 9/x^2}{x^2 + \frac{81}{x^2}}\, dx$

$$\Rightarrow I = \int \frac{1 + 9/x^2}{x^2 + \left(\frac{9}{x}\right)^2 - 18 + 18}\, dx$$

$$= \int \frac{1 + 9/x^2}{\left(x - \frac{9}{x}\right)^2 + 18}\, dx$$

Let $x - \frac{9}{x} = t \Rightarrow \left(1 + \frac{9}{x^2}\right) dx = dt$

$$\therefore\ I = \int \frac{dt}{t^2 + 18} \Rightarrow I = \int \frac{dt}{t^2 + \left(3\sqrt{2}\right)^2}$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}} \tan^{-1}\left(\frac{t}{3\sqrt{2}}\right) + c$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 9}{3\sqrt{2}x}\right) + c$$

OR

Let $I = \int \underset{\text{I}}{x^2} \underset{\text{II}}{\sin 2x}\, dx$

$$= x^2\left(\frac{-\cos 2x}{2}\right) - \int 2x \cdot \left(\frac{-\cos 2x}{2}\right) dx$$

$$= \frac{-1}{2} x^2 \cos 2x + \int \underset{\text{I}}{x} \underset{\text{II}}{\cos 2x}\, dx$$

$$= \frac{-1}{2} x^2 \cos 2x + \left[x\left(\frac{\sin 2x}{2}\right) - \int \frac{\sin 2x}{2}\, dx\right]$$

$$= \frac{-1}{2} x^2 \cos 2x + \frac{x \sin 2x}{2} + \frac{1}{4} \cos 2x + c$$

$$\therefore\ I = \frac{-x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{\cos 2x}{4} + c$$

16. Given $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$

We have $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c} \Rightarrow |\vec{a} + \vec{b}|^2 = |-\vec{c}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = |\vec{c}|^2$

$\Rightarrow 9 + 25 + 2|\vec{a}||\vec{b}| \cos\theta = 49$

$\Rightarrow$ 2 × 3 × 5 × cosθ = 49 – 34 = 15

$\Rightarrow \cos\theta = \frac{15}{30} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} = 60°$

17. Let A be the event that the bulb is defective.

$\therefore$ $P(E_1) = \frac{50}{100}$, $P(E_2) = \frac{25}{100}$, $P(E_3) = \frac{25}{100}$

$P(A/E_1) = \frac{4}{100}$, $P(A/E_2) = \frac{4}{100}$, $P(A/E_3) = \frac{5}{100}$

$\therefore$ Required probability,

$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$

$$= \frac{50}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{5}{100}$$

$$= \frac{200 + 100 + 125}{10000} = \frac{425}{10000} = \frac{17}{400}$$

18. Given planes are

$x - cy - bz = 0$ ...(i)

$cx - y + az = 0$ ...(ii)

$bx + ay - z = 0$ ...(iii)

The d.r.'s of normal to plane (i), (ii) and (iii) are (1, $-c$, $-b$), (c, –1, a) and (b, a, –1) respectively.

All planes pass through same line, then the line is perpendicular to each of the three normals.

The d.r's. of line from planes (i) and (ii) are

$$\begin{vmatrix} -c & -b \\ -1 & a \end{vmatrix}, -\begin{vmatrix} 1 & -b \\ c & a \end{vmatrix}, \begin{vmatrix} 1 & -c \\ c & -1 \end{vmatrix}$$

i.e., $-ac - b$, $-a - bc$, $-1 + c^2$ ...(iv)

The d.r.'s of line from planes (ii) and (iii) are

$$\begin{vmatrix} -1 & a \\ a & -1 \end{vmatrix}, -\begin{vmatrix} c & a \\ b & -1 \end{vmatrix}, \begin{vmatrix} c & -1 \\ b & a \end{vmatrix}$$

i.e., $1 - a^2, c + ab, ac + b$ ...(v)

The d.r.'s in (iv) and (v) are in proportion, then

$$\frac{-ac-b}{1-a^2} = \frac{-a-bc}{c+ab} = \frac{-1+c^2}{ac+b}$$

$$\Rightarrow \frac{-ac-b}{1-a^2} = \frac{-a-bc}{c+ab}$$

$\Rightarrow ac^2 + bc + a^2bc + ab^2 = a + bc - a^3 - a^2bc$

$\Rightarrow c^2 + abc + b^2 = 1 - a^2 - abc$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

OR

The equation of a plane passing through (2, 2, –1) is
$a(x - 2) + b(y - 2) + c(z + 1) = 0$ ...(i)

This plane also passes through (3, 4, 2).

$\therefore\ a(3 - 2) + b(4 - 2) + c(2 + 1) = 0$

$\Rightarrow a + 2b + 3c = 0$ ...(ii)

Now, plane (i) is parallel to the line whose direction ratios are 7, 0, 6

Therefore, $7a + 0(b) + 6c = 0$ ...(iii)

Solving (ii) and (iii) by cross-multiplication method, we get

$$\frac{a}{(2)(6)-(0)(3)} = \frac{b}{(7)(3)-(6)(1)} = \frac{c}{(0)(1)-(2)(7)}$$

$$\Rightarrow \frac{a}{12} = \frac{b}{15} = \frac{c}{-14} = \lambda \text{ (say)}$$

$\Rightarrow a = 12\lambda, b = 15\lambda, c = -14\lambda$

Substituting the values of a, b, c in (i), we get

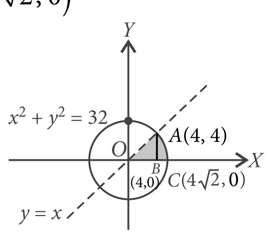
$12\lambda(x - 2) + 15\lambda(y - 2) - 14\lambda(z + 1) = 0$

$\Rightarrow 12x - 24 + 15y - 30 - 14z - 14 = 0$ $[\because \lambda \neq 0]$

$\Rightarrow 12x + 15y - 14z = 68$, which is the required equation of plane.

19. The given equation of the circle is $x^2 + y^2 = 32$ and the line is $y = x$

These intersect at $A(4, 4)$ in the first quadrant. The required area is shown shaded in the figure. Points $B(4, 0)$ and $C\left(4\sqrt{2}, 0\right)$



$\therefore$ Required area = Area $BACB$ + Area $OABO$

$$= \int_4^{4\sqrt{2}} y_1 dx + \int_0^4 y_2 dx = \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx + \int_0^4 x\,dx$$

$$= \int_4^{4\sqrt{2}} \sqrt{\left(4\sqrt{2}\right)^2 - x^2}\,dx + \int_0^4 x\,dx$$

$$= \left[\frac{x\sqrt{32-x^2}}{2} + \frac{32}{2}\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}} + \left[\frac{x^2}{2}\right]_0^4$$

$$= \frac{4\sqrt{2}\times 0}{2} + 16\sin^{-1}1 - \left(\frac{4\times 4}{2} + 16\sin^{-1}\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(4^2 - 0)$$

$$= 16\cdot\frac{\pi}{2} - \left(8 + 16\cdot\frac{\pi}{4}\right) + 8 = 16\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 4\pi \text{ sq.units}$$

OR

We have curves $x - y + 2 = 0$ and $x = \sqrt{y}$.

$x = \sqrt{y} \Rightarrow y = x^2$, which is a parabola with vertex at origin.

From the given equations, we get

$x - x^2 + 2 = 0$

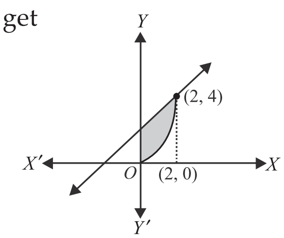
$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = 2$ or $x = -1$

$\Rightarrow x = 2$

$[\because x \neq -1, x$ is positive$]$

When $x = 2, y = 4$



So, the point of intersection is (2, 4).

$$\therefore \text{ Required area } = \int_0^2 (x+2)dx - \int_0^2 x^2 dx$$

$$= \int_0^2 (x + 2 - x^2)dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2 = 2 + 4 - \frac{8}{3} = \frac{10}{3} \text{ sq. units}$$

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